**Purpose**
In this two-day lesson, students will model temperature data. They will use “known temperature stations” in order to estimate temperatures at any given point accurately. Websites that give the temperature at a specific place typically do not give the actual values; they give an estimate based on meteorological data. Explain to students that temperatures are not measured everywhere and educated estimates need to be made. Have the students imagine they are meteorologists interested in making a model to estimate temperature at a given time and at a given location.

**Prerequisites**
Students need to understand ratios and equations in one variable, as the lesson is heavily dependent on these areas. Additionally, reading, interpreting, and understanding graphs is important for completing the lesson.

**Materials**
*Required:* Rulers or straightedges.
*Suggested:* Graphing paper or a graphing utility.
*Optional:* (For three-dimensional models) Cardboard, sticks or drinking straws, and scissors.

**Worksheet 1 Guide**
The first three pages of the lesson constitute the first day’s work. Students are asked to estimate the temperature at a point on a map between two other points where the temperature has been measured. It is important that students understand that the diagrams given are drawn to scale. This fact should arise from discussion about variable identification in questions 1 and 2. The students should begin to formulate ideas about linearity. Questions 4 and 5 ask students to extend their model when the unknown points do not fall in a straight line with two known temperature stations. There will be a variety of solution methods, but each should use the concept of linearity or a constant rate of change between two points.

**Worksheet 2 Guide**
The fourth and fifth pages of the lesson constitute the second day’s work. Students are first given a definition of a linear function and then questions have students making connections between their Day 1 models and the graph of a linear function. Then students will give their description of the meaning of average rate of change and its relation to linear functions. They will be challenged to calculate the rate of change of a linear function.

**CCSSM Addressed**
A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
F-IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
When you look on websites such as www.weather.com to find out the current temperature, you usually don’t get the actual measured temperature for your town — it’s an educated estimate!

**Leading Question**
How would you create a model like one a meteorologist would use to estimate the temperature?
1. How would you expect temperatures to change in between two towns? Use your experience to make an educated guess. Would you expect the temperature to change gradually or suddenly? Explain.

- 62° F
- 65° F
- 66° F
- 68° F

2. How can you estimate the change in temperature between two towns? Use your ideas from above to estimate the temperature in between those towns with known temperature. Show your work.

- 53° F
- 7° F
- 47° F

3. Describe a mathematical model for estimating the temperature in a given town between two towns for which the temperatures are known. Write your description in words first, then in mathematical symbols.
4. How would you estimate the temperature in a town that isn’t between two towns where the actual temperatures are measured?

- 79°F
- 72° F
- 76° F
- ?° F

5. It is a cold, rainy day. You and your friends want to drive to an indoor skate park (S) from home (H). Your parents are worried that it will get colder as you get closer to the skate park and the rain will freeze; you’re not allowed to drive if there’s a chance of sleet or snow. Use the map below to determine if it’s safe to go.

- 33°F
- 30°F
- 35°F
- 36°F
- H
- S

6. Describe, verbally and mathematically, your model for estimating the temperature in any given town. Do you think your model will always work? Are there factors that you didn’t consider that professional meteorologists probably use in their own models?
Using functions is one way to model the change in temperature. A linear function is a function that grows by equal distances over equal intervals. The amount that they change is called the slope and is usually denoted by \( m \).

7. Is there a way that you can use a graph to represent the temperatures in the towns shown in question 1? Plot the towns as points on the coordinate plane. Draw a line containing the points.

8. Use the method above to graph and represent the situation from question 2. What are the coordinates of the middle point? Does this coordinate have any relationship with the temperature you estimated?
9. Modify your method for modeling the situation in question 4 by using graphs of linear functions. Does this model give the same result as in question 4? How is the rate of change in the temperature between two points described on the graph?

You may need to use more than one graph.

10. Describe how you used graphs of linear functions to model estimating temperatures at given points. What are the similarities and differences between your original model and the linear function model?

11. Use the work you've done to describe what is meant by “rate of change”. How does it relate to the graph of a linear function? Is there a way to calculate or estimate the rate of change of a linear function easily?
ESTIMATING TEMPERATURES
Teacher’s Guide — Possible Solutions

The solutions shown represent only some possible solution methods. Please evaluate students’ solution methods on the basis of mathematical validity.

1. The temperatures here would seem to indicate that temperature changes continuously and constantly over intervals of equal length. That is, temperature appears to change linearly.

2. The only variable affecting the temperature, as far as we can see, is distance. Many other variables affect temperature, but those data are not given here. The student may be able to refine the model to include those variables later, if necessary. The temperature of the unknown is approximately 49°F.

3. The temperature changes at the same rate over equal distances.

Let $a$, $b$, and $x$ represent the temperature in degrees at points $A$, $B$, and $X$, respectively. If an unknown temperature point, $X$, lies between (collinearly with) two known temperature points, $A$ and $B$ where $A$ is the lower temperature, then $x = (AX/AB)(b - a) + a$.

4. The model found in question 3 can be used twice. First, construct a line between any of the two known points (the line between 79°F and 76°F is shown). Second, construct a line through the last known point and the unknown point. Use the model to estimate the temperature at the point of intersection of the two lines. Finally, use that estimation to estimate the temperature at the desired point.

5. The model from question 4 can be used with any 3 known points. Students should find that different sets of known points produce different answers. They may conclude that the set of closest known points should be used or that the average of the answers for all sets of three known points should be used.

6. A model description is given in the solution to question 4. The topography of the area is one major variable that has been left out of the model. Hills and valleys affect the flow of air and, hence, temperatures.

7. The answers to the previous questions are replicated in the context of a linear graph.

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10. Linear functions describe the rate of change (in their slope) of the temperature. The distance is the $x$-value and the temperature is the $y$-value.

11. For a linear function, the slope is the rate of change.
Suppose you wanted to estimate a temperature outside the intervals in questions 1 and 2. What would you do? Try an example in which the numbers are not just monotone, but with the perturbation of some “noise”. (What might cause such “noise”? Changes in elevation, wind, etc.) The basic pattern still looks linear. Fit a line to the data as well as you can. This idea can lead to the method of least squares.

No two of the three given temperature stations have the same temperature. Therefore, one of the three numbers must be between the other two. In this case, 76°F is between 72°F and 79°F. Where on the line between 72°F and 79°F is the temperature also 76°F? Find that point and connect it by a straight line to the vertex with temperature 76°F. All lines of constant temperature will be parallel to this one. Fill in these lines for all whole-number temperatures between 72°F and 79°F. Now estimate the temperature at the point marked ?°F. If two of the original three temperatures were the same, how would you modify the procedure you just found? What is now the direction of lines of constant temperature?

If the point marked ?°F were outside the triangle, how would you estimate its temperature? Draw the points with temperature 72°F, 76°F, and 79°F on a flat surface, and construct a vertical post of heights 2, 6, and 9 (ignoring the 7) at each of these points. Lay a flat surface on top of these three posts. How does the height of the point marked ?°F compare with the height you estimated before? Draw lines of constant height onto your surface. How do they compare with the lines you drew before?

You now have four points whose temperature are known. Take any three of these points and use them to estimate the temperature at S as you did above. Use a different set of three points and do it again. How many such sets of three points are there? Look at the guesses for the temperature at S that you now have. Are they equal? If not, order them. Can you convince your parents that the temperature will be between the highest and the lowest of these? How do you feel about the average of the four?